DEPARTMENT OF SCIENCES
PROGRAM - MATHEMATICS

organises an international
webinar on

AN INTRODUCTION TO WAVELETS

on September 25, 2023
Monday
2:30-3:30 PM, IST

PROFESSOR MARIA SKOPINA
DEPARTMENT OF HIGHER MATHEMATICS
ST. PETERSBERG UNIVERSITY, RUSSIA

Registration link: https://forms.gle/Ezc1NCCXqkZhnguu6
Meeting Link: https://tinyurl.com/bd489hcv

E-certificates will be provided to all the participants

TITLE - An International Webinar on "Introduction to Wavelets".

Date: 25th September 2023, 2:30 PM

Venue: Online Mode, Microsoft Teams

Resource Person: **Prof. Maria Skopina**, Department of Higher Mathematics, St. Petersburg University, Russia.

No. of Participants: 33 Students

The Department of Sciences, Program - Mathematics, Manav Rachna University organized an International Webinar on Introduction to Wavelets on September 25, 2023. Prof. Maria Skopina, Department of Higher Mathematics St. Petersburg University, Russia was the Guest Speaker.

Prof. Maria is DSc. in Mathematics (2000) from St. Petersburg, Department of Steklov, Mathematical Institute, RAS and Ph.D. in Mathematics (1980) from Leningrad (St. Petersburg) State University. Her Research areas include Wavelets, Fourier analysis, approximation theory, p-adic harmonic analysis, sampling theory. She has authored more than 80 research papers & 2 books in AMS & Springer. She has diverse teaching experience as visiting professor at Universities in Brazil, USA., Ukraine, Iran etc. She has guided many Ph.D. & D.Sc. students as well. Prof. Skopina has served as Chairman, Vice chairman, Advisory board member in various Editorial boards of journals, societies, conferences etc. She has been the recipient of many Research grants from the Russian Foundation for Basic Research since 1993 till date.

Ms. Anshika Vashisht, M.Sc. Mathematics 3rd semester student started the session and welcomed the guest & audience. Then Dr. Aparna Vyas, Program Head-Mathematics, introduced the guest speaker to the audience.

Prof. Maria briefly introduced the term wavelets and its applications. Wavelets are mathematical functions that cut up data into different frequency components, and then study each component with a resolution matched to its scale. They have advantages over traditional Fourier methods in analyzing physical situations where the signal contains discontinuities and sharp spikes. The wavelet approximation technique is a recent tool to detect and analyze abrupt change in seismic signal processing. Prof. In mathematical analysis, and especially in real and harmonic analysis, an Orlicz space is a type of function space which generalizes the Lp spaces. She detailed wavelet, bio-orthogonal wavelets frames, multi- wavelet systems, Bessel function and matrix existential function. She briefly explained all the basic notions and terminology with examples. She also covered the applications of wavelets in many areas like science and engineering; particularly, wavelets are very successfully used in image analysis, communication systems, biomedical imaging, radar, air acoustics, theoretical mathematics, control systems, signal analysis for waveform representation and segmentation, time-frequency analysis and fast algorithms for easy implementation.

The overall talk enriched the students and faculty with the current Applications of Wavelet Approximation. Prof. Maria humbly and satisfactorily answered the queries. More than 40 participants including faculty members attended the session and showed interest in the interactive session.

At the end, the session was concluded with a vote of thanks by Ms. Savitta Saini.

Introduction
MRA and Scaling Functions
Wavelet Systems

Let φ be a solution of the refinement equation

$$\hat{\varphi}(\xi) = m_0(\xi/2)\hat{\varphi}(\xi/2).$$

Then for any positive integer n, we have

$$\hat{\varphi}(\xi) = \hat{\varphi}(\xi/2^n)\prod_{j=1}^{n} m_0(\xi/2^j).$$

If $\hat{\varphi}$ is continuous at 0 and $\hat{\varphi}(0) = 1$, then $m_0(0) = 1$. Hence

$$\hat{\varphi}(\xi) = \prod_{j=1}^{\infty} m_0(\xi/2^j),$$

where the infinite product converges for any ξ

Short Course of Wavelets

Скопина Мария Александровна

Introduction   Wavelet Functions
MRA and Scaling Function   Examples
Wavelet Sistems   Computational algorithms

Let us now consider the inverse problem: to reconstruct a function f by the coefficients of its wavelet expansion. As before, we identify the function f with the sequence $\{c_{j_0 n}\}_{n\in\mathbb{Z}}$. Thus we have to find the elements of this sequence for the initial data $c_{0,k}$, $d_{j,k}$, $k\in\mathbb{Z}$, $j = 0,\ldots, j_0$. Let $j < j_0$. Using the above notation for $\tilde{f}$, we have

$$\tilde{f} = \sum_{k\in\mathbb{Z}} c_{jk}\varphi_{jk} + \sum_{i=j}^{j_0}\sum_{k\in\mathbb{Z}} d_{ik}\psi_{ik}.$$

Taking the scalar product of both sides of this equality and $\tilde{\varphi}_{j+1,n}$, we get

$$c_{j+1,n} = \sum_{k\in\mathbb{Z}} c_{jk}\langle\varphi_{jk},\tilde{\varphi}_{j+1,n}\rangle + \sum_{k\in\mathbb{Z}} d_{jk}\langle\psi_{jk},\tilde{\varphi}_{j+1,n}\rangle.$$

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Let $\{V_j\}_{j\in\mathbb{Z}}$ be an MRA with the scaling function φ, whose integer shifts form an orthonormal system.

Denote by W_j the orthogonal complement to V_j in the space V_{j+1}. By the properties of the Hilbert space, V_{j+1} can be decomposed into the direct sum $V_{j+1} = V_j \oplus W_j$. Thus we obtain the collection $\{W_j\}_{j\in\mathbb{Z}}$ of subspaces of the space $L_2(\mathbb{R})$ such that $W_j \perp V_j$, $W_j \perp W_k$ for all $j, k \in \mathbb{Z}$, $k \neq j$ and

$$V_j = V_k \oplus W_k \oplus \cdots \oplus W_{j-1}$$

for all $j, k \in \mathbb{Z}$, $k < j$. In view of **MR2** and **MR3**, it follows that

$$L_2(\mathbb{R}) = V_0 \oplus W_0 \oplus W_1 \oplus \ldots,$$

$$V_0 = W_{-1} \oplus W_{-2} \oplus \ldots \ldots$$

Combining these two decompositions, we get

$$L_2(\mathbb{R}) = \bigoplus_{j=-\infty}^{\infty} W_j \qquad (11)$$

Short Course of Wavelets