MANAV RACHNA
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MANAV RACHNA UNIVERSITY

(FORMERLY MANAV RACHNA COLLEGE OF ENGINEERING
NAAC ACCREDITED 'A' GRADE INSTITUTION)

Declared as State Private University under section 2f of the UGC act, 1956

DEPARTMENT OF MATHEMATICS

"T3Examination, May2017-18"

Semester: 4th
Subject: Group theory
Branch: Maths
Course Type: Core
Time: 3 Hours
Max.Marks: 80

Date of Exam: 17/05/2018
Subject Code: MAH227-T
Session: II
Course Nature: Hard
Program: B.Sc
Signature: HOD/Associate HOD: Amit Awasthi

Note: All questions are compulsory from part A (2*10 = 20 Marks). Attempt any two questions from Part B (15 Marks each). Attempt any two Questions from Part -C (15 Marks each).

PART -A

Q.1 (a) Define Normal Sub group.

(b) Is Z3⊕Z9 is isomorphic to Z27?

(c) Define Automorphism of groups.

(d) Define Conjugate elements in Group theory.

(e) Show that the mapping $f: I \rightarrow I$ defined by f(x) = -x for all $x \in I$ is Automorphism of (I,t).

(f) Show that if $0(Aut\ G) > I\ then\ o(G) > 2$.

(g) Prove that every sub group of an abelian group is always normal.

(h) Show that every quotient group of an abelian group is abelian.

(i) Prove that Normalizer of $a \in G$ is a sub group of G.

(j) State Cauchy Theorem for finite Abelian groups.

PART - B

Q.2 (a) If $f: G \rightarrow G'$ is a homomorphism, then prove that Kernel of f is a normal subgroup of G. (6)

(b) Prove that every homomorphic image of a group. G is isomormorphic to some quotient group of G. (9)

Q.3 (a) Prove that every finite group is isomorphic to a permutation group. (8)

(b) If $f: G_1 \rightarrow G_2$ and f is homomorphism such that e_1 is Identity of G_1 and e_2 is Identity of G_2. Then prove that $f(e_1) = e_2$.

Q.4 (a) If M and N are two Normal subgroups of Gs.t.M N . Then Prove that $\frac{G}{N} \cong \frac{G/M}{N/M}$. (8)

(b)Prove that homomorphism $f: G \rightarrow G'$ is an Isomorphism iff $Ker\ f = \{e\}$where e the Identity of G. (7)

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PART -C

Q.5 (a) If G is finite abelion group of order n and m is +ve integer such that (m,n) = 1. Then show that $f: G \rightarrow G$ defined by $f(x) = x^m$ is an Automorphism. (6)

(b) Prove that the set Inn(G) of all inner Automorphism of a gp. G is a normal sub group of group Aut(G) as its automorphism. (9)

Q.6 (a) Prove that an abelian group of order 10 is cyclic. (7.5)

(b) Prove that $Z_2 \oplus Z_2 \oplus Z_2$ has seven sub groups of order 2. (7.5)

Q.7(a) Prove that a group G is internal direct product of two sub groups H_1 and H_2 iff H_1 and H_2 are normal subgroups of G. (8)

(b)Prove that the Identity mapping is only inner automorphism for an abelian group. (7)

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