MANAV RACHNA vidyanatariksha
MANAV RACHNA UNIVERSITY
(FORMERLY MANAV RACHNA COLLEGE OF ENGINEERING
NAAC ACCREDITED 'A' GRADE INSTITUTION)
Declared as State Private University under section 2f of the UGC act, 1956

# DEPARTMENT OF MATHEMATICS

*"T3Examination, May 2017-18"*

**Semester**:4th
**Subject**:Advanced Analysis
**Branch**: Maths
**Course Type:**Core
**Time**: 3 Hours
**Max.Marks**: 80

**Date of Exam**:21/05/2018
**Subject Code**:MAH226-T
**Session**: II
**Course Nature:**Hard
**Program: B.Sc**
**Signature: HOD/Associate HOD:**

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Note: All questions are compulsory from part A (2*10 = 20 Marks). Attempt any two questionsfrom Part B (15 Marks each). Attempt any two Questions from Part -C (15 Marks each).

## PART -A

Q.1 (a) Write the polar form of the complex number $-1-i$.

(b) Define Analytic Function.

(c) Write Cauchy Riemann Equations in polar form.

(d) Separate in to real and imaginary parts of the function $\sin(x+iy)$.

(e) Define improper integral. Give examples of different types of improper integrals.

(f) Examine the convergence of $\int_0^\infty \frac{dx}{1+x^2}$.

(g) State Dirichlet's Test for convergence of Improper Integrals.

(h) Discuss the convergence of $\int_0^\infty \sqrt{x}\, e^{-x} dx$.

(i) Define single valued and multiple valued function, with the help of suitable examples.

(j) State comparison tests for Improper Integrals.

## PART - B

Q.2 (a) Examine the convergence of the integral $\int_0^\infty \cos x^2\, dx$. (8)

(b) Examine the convergence of the integral $\int_a^b \frac{dx}{(x-a)\sqrt{b-x}}$ (7)

Q.3 (a)State and prove Abel's Test for convergence of improper integrals. (8)

(b) Discuss the convergence of the Beta function. (7)

Q.4 (a) Using Dirichlet's test, show that $\int_0^\infty \frac{\sin x}{x} dx$is convergent. (8)

(b)Using the concept of term by term differentiation discuss the uniform convergence of $f_n(x) = nxe^{-nx^2}$

(7)

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## PART -C

Q.5 (a) Show that the function

$$f(z) = \begin{cases} \dfrac{x^3(1+i) - y^3(1-i)}{x^2+y^2}, & \text{when } z \neq 0 \\ 0 \quad , & \text{when } z = 0 \end{cases}$$

Is continuous and C.R. equations are satisfied at the origin, but not analytic at origin. (8)

(b) Prove that an analytic function with constant modulus is constant. (7)

Q.6 (a) If $u + v = \frac{\sin 2x}{\cosh 2y - \cos 2x}$ and $f(z) = u + iv$ is an analytic function of z, Then find f(z). (8)

(b) If $\omega = \emptyset + i\varphi$ represents the complex potential for an electric field and $\varphi = (x^2 - y^2) + \frac{x}{x^2+y^2}$ ,

Determine the function $\emptyset$. (7)

Q.7(a) State and prove necessary and sufficient condition for a function $f(z) = u + iv$ to be analytic. (10)

(b) Prove that real and imaginary parts of an analytic function are Harmonic functions. (5)

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